Such a model in the equinoctial elements is given in Refs. 15–17. Green[15] considered the effect of holding the moon's position constant during one satellite orbit. He noted that the major impact is in the short-periodic motion. While Ref. 1 emphasizes the moon's motion as a source of second-order coupling, there is also the possibility of coupling between the J_2 secular effects and the shallow tesseral resonance oscillations that occur in near-geosynchronous orbits.

Finally, a double-averaging theory (see Collins[18]) is also an appropriate tool for analyzing the very long-term motion of near-geostationary orbits.[19]

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Reply by Author to P. Cefola

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THE author is grateful to Dr. Cefola for his keen interest in Ref. 1 and for pointing out the connection between the results of Ref. 1 and other formulations. It is unfortunate that essentially all references quoted by Dr. Cefola are either conference papers or internal notes, which limits their dissemination considerably. Perhaps it would be of interest to summarize their principal results in an archival publication.

When Ref. 1 is compared with other results, it should not be overlooked that the scope of Ref. 1 was limited to providing useful results for just one specific, but extremely important, class of orbits. No attempt at generalization of the results of Ref. 1 was made. It is believed that a practicing engineer dealing with geostationary orbits would prefer the explicit closed-form results of Ref. 1 to the more general but recursive formulation suggested by Dr. Cefola.

Concerning the use of the equinoctial elements, it is felt that proper credit was given to earlier work by referring to the archival publication by Broucke and Cefola (Ref. 12 in Ref. 1). The derivation of Eqs. (3) in Ref. 1 was, in fact, carried out independently using Campbell's formulation (details are described in an internal ESOC report[2]).

The form of Dr. Cefola's general result for the averaged third-body potential, as described in his Eqs. (1) and (2), shows complete agreement with Eqs. (18–20) of Ref. 1. Whereas the recursive code, as advocated by Dr. Cefola, would offer advantages from an overall generality and flexibility of point of view, the explicit results of Ref. 1 are of more practical value for the specific case of a near-geostationary orbit. The remark by Dr. Cefola that the Poisson series analysis must be revisited each time when the orbital-type changes should be seen in the same light.

The comments by Dr. Cefola on the zonal and tesseral harmonics formulation are of the same nature as those on the potential development and can therefore be answered by the same argument.

The procedure for obtaining initial conditions for the mean elements that was adopted in Ref. 1 is rather straightforward since it was needed only for establishing the accuracy of the long-term model and not for an accurate orbit prediction. The improvements in this procedure suggested by Dr. Cefola could have a slightly beneficial effect on the accuracies quoted in Table 3 of Ref. 1.

Finally, it is noted that the verification of the coupling between J_2 secular effects and shallow tesseral resonances would require a number of controlled simulation runs and cannot be commented on now.

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Received Aug. 5, 1986.

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